

# NNLO QCD corrections to event shape variables in electron positron annihilation

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**Abstract.** Precision studies of QCD at  $e^+e^-$  colliders are based on measurements of event shapes and jet rates. To match the high experimental accuracy, theoretical predictions to next-to-next-to-leading order (NNLO) in QCD are needed for a reliable interpretation of the data. We report the first calculation of NNLO corrections ( $\mathcal{O}(\alpha_s^3)$ ) to three-jet production and related event shapes, and discuss their phenomenological impact.

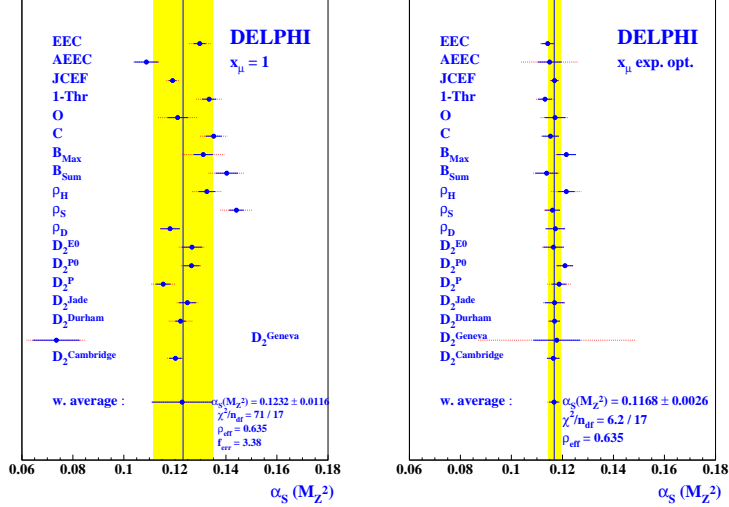
## 1. Introduction

Measurements at LEP and at earlier  $e^+e^-$  colliders have helped to establish QCD as the theory of strong interactions by directly observing gluon radiation through three-jet production events. The LEP measurements of three-jet production and related event shape observables are of a very high statistical precision. The extraction of  $\alpha_s$  from these data sets relies on a comparison of the data with theoretical predictions. Comparing the different sources of error in this extraction, one finds that the purely experimental error is negligible compared to the theoretical uncertainty. There are two sources of theoretical uncertainty: the theoretical description of the parton-to-hadron transition (hadronisation uncertainty) and the uncertainty stemming from the truncation of the perturbative series at a certain order, as estimated by scale variations (perturbative or scale uncertainty). Although the precise size of the hadronisation uncertainty is debatable and perhaps often underestimated, it is certainly appropriate to consider the scale uncertainty as the dominant source of theoretical error on the precise determination of  $\alpha_s$  from three-jet observables.

The three-jet rate and event shapes related to it can be expressed in perturbative QCD by dimensionless coefficients. These coefficients depend either on the jet resolution parameter or on the event shape variable. Typically, one denotes these coefficients by  $A, B, C, \dots$  at LO, NLO, NNLO, etc.

The perturbative expansion for the distribution of a generic observable  $O$  up to NNLO for renormalisation scale  $\mu^2 = s$  and  $\alpha_s \equiv \alpha_s(s)$  is given by

$$\frac{1}{\sigma_0} \frac{d\sigma}{dO} = \left( \frac{\alpha_s}{2\pi} \right) \frac{dA}{dO} + \left( \frac{\alpha_s}{2\pi} \right)^2 \frac{dB}{dO} + \left( \frac{\alpha_s}{2\pi} \right)^3 \frac{dC}{dO}. \quad (1)$$



**Figure 1.** Results of a NLO QCD fit to 18 hadronic observables for (a) a fixed renormalisation scale  $\mu = \sqrt{s}$  and (b) optimising the scale choice to fit the data. The figures are taken from Ref. [4]

normalised to the tree-level cross section for  $e^+e^- \rightarrow q\bar{q}$ . It can easily be related to the physical observable  $\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dO}$  using the known relation between  $\sigma_{\text{had}}$  and  $\sigma_0$ .

Ignoring the numerically negligible singlet contribution,  $A$ ,  $B$  and  $C$  depend only on the jet resolution parameter or the event shape variable under consideration, and are independent of electroweak couplings, centre-of-mass energy and renormalisation scale.

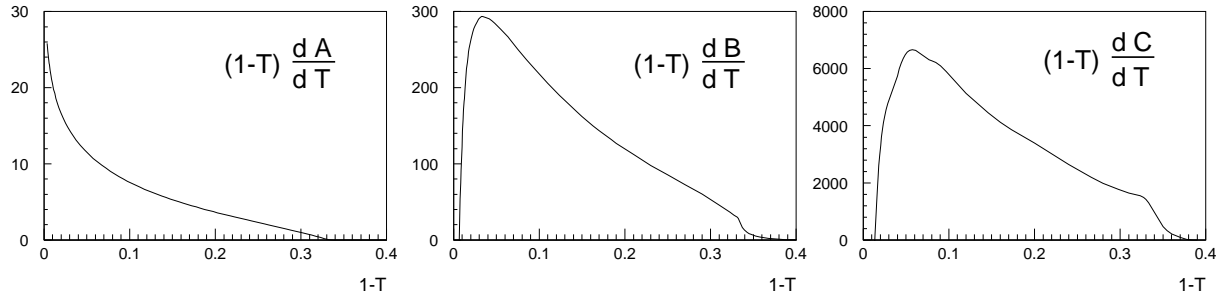
QCD studies of event shape observables at LEP [1] based around the use of fixed-order NLO parton-level event generator programs [2] have shown that the current error on  $\alpha_s$  from these observables [3] is dominated by the theoretical uncertainty. To illustrate this, the DELPHI collaboration made fits to 18 hadronic observables (a) at fixed renormalisation scale,  $\mu = \sqrt{s}$  and (b) by treating the renormalisation scale as a free parameter in the fit. The results are displayed in Fig. 1, taken from Ref. [4]. With a fixed scale (fig 1(a)), the value of the strong coupling extracted varies considerably amongst leading to a value of  $\alpha_s(M_Z) = 0.1232 \pm 0.0116$ . On the other hand, fig 1(b) shows the result of letting the scale vary (and thereby estimating the uncalculated higher order corrections), and leads to a more consistent fit and a much smaller error  $\alpha_s(M_Z) = 0.1168 \pm 0.0026$ . Clearly, to improve the determination of  $\alpha_s$ , the calculation of the NNLO corrections to these observables becomes mandatory.

## 2. Thrust distribution

As an example, we focus on the thrust distribution,  $O = T$ . The LO and NLO coefficients  $A(T)$  and  $B(T)$  are displayed for comparison in Figure 2. In the numerical evaluation, we use  $M_Z = 91.1876$  GeV and  $\alpha_s(M_Z) = 0.1189$  [3].

The NNLO coefficient  $C(T)$  has been recently computed [5]. It is based on two-loop  $\gamma^* \rightarrow q\bar{q}g$  matrix elements [6, 7], one-loop four-parton matrix elements [8] and tree-level five-parton matrix elements [9].

The two-loop  $\gamma^* \rightarrow q\bar{q}g$  matrix elements were derived in [6] by reducing all relevant Feynman integrals to a small set of master integrals using integration-by-parts [10] and Lorentz invariance [11] identities. The master integrals [12] were computed from their differential equations [11] and expressed analytically in terms of one- and two-dimensional harmonic



**Figure 2.** Coefficients of the leading order, next-to-leading order and next-to-next-to-leading order contributions to the thrust distribution.

polylogarithms [13].

The one-loop four-parton matrix elements relevant here [8] were originally derived in the context of NLO corrections to four-jet production and related event shapes [14, 15].

The four-parton and five-parton contributions to three-jet-like final states at NNLO contain infrared real radiation singularities, which have to be extracted and combined with the infrared singularities present in the virtual three-parton and four-parton contributions to yield a finite result. In our case, this is accomplished by introducing antenna subtraction functions [16, 17], which encapsulate all singular limits due to the emission of one or two unresolved partons between two colour-connected hard partons, and are sufficiently simple to be integrated analytically [18].

The resulting numerical programme, **EERAD3**, yields the full kinematical information on a given multi-parton final state. It can thus be used to compute any infrared-safe observable related to three-particle final states at  $\mathcal{O}(\alpha_s^3)$ . The NNLO coefficient  $C(T)$  for the thrust distribution obtained with **EERAD3** is shown in Fig. 2 [5].

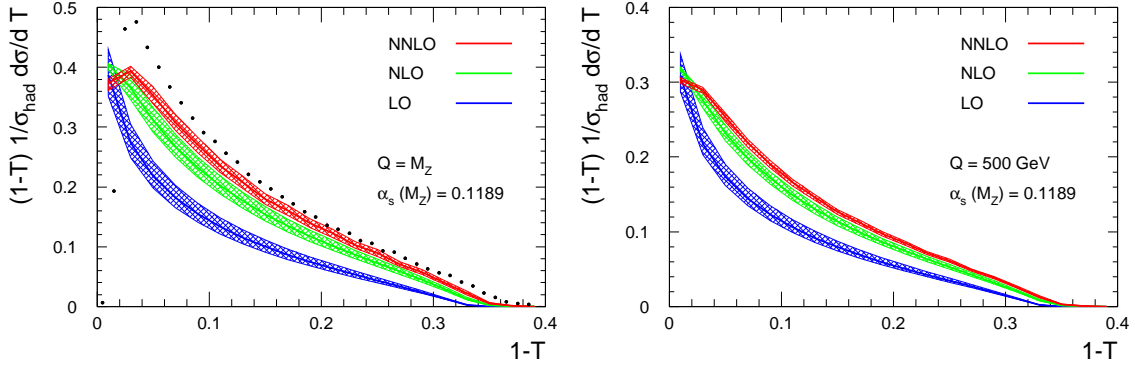
Figure 3 displays the perturbative expression for the thrust distribution at LO, NLO and NNLO, evaluated for LEP and ILC energies. The error band indicates the variation of the prediction under shifts of the renormalisation scale in the range  $\mu \in [Q/2; 2Q]$  around the  $e^+e^-$  centre-of-mass energy  $Q$ .

It can be seen that even at linear collider energies, inclusion of the NNLO corrections enhances the thrust distribution by around 10% over the range  $0.03 < (1-T) < 0.33$ , where relative scale uncertainty is reduced by about 30% between NLO and NNLO. Outside this range, one does not expect the perturbative fixed-order prediction to yield reliable results. For  $(1-T) \rightarrow 0$ , the convergence of the perturbative series is spoilt by powers of logarithms  $\ln(1-T)$  appearing in higher perturbative orders, thus necessitating an all-order resummation of these logarithmic terms [19, 20], and a matching of fixed-order and resummed predictions [21].

The perturbative parton-level prediction is compared with the hadron-level data from the ALEPH collaboration [22] in Figure 3. The shape and normalisation of the parton level NNLO prediction agrees better with the data than at NLO. We also see that the NNLO corrections account for approximately half of the difference between the parton level NLO prediction and the data.

### 3. Conclusions

We developed a numerical programme which can compute any infrared-safe observable through to  $\mathcal{O}(\alpha_s^3)$ , which we applied here to determine the NNLO corrections to the thrust distribution. These corrections are moderate, indicating the convergence of the perturbative expansion. Their inclusion results in a considerable reduction of the theoretical error on the thrust distribution and will allow a significantly improved determination of the strong coupling constant from jet



**Figure 3.** Thrust distribution at LEP and at the ILC with  $Q = 500$  GeV.

observables from existing LEP data.

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